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# Accurate and Fast Algorithms for Dense Motion Estimation Based on Global Energy Minimisation

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#### joint work with

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# Introduction (1)

# What is the Goal?

### Given

• image sequence  $f(x_1, x_2, x_3)$ 

| location | $(x_1,x_2)$ | $\in$ | $oldsymbol{\Omega}$ |
|----------|-------------|-------|---------------------|
| time     | $x_3$       | $\in$ | [0,T]               |

#### Wanted

• interframe displacement field  $\mathbf{u} = (u_1, u_2, 1) \rightarrow$  optic flow



# Introduction (1)

# What is the Goal?

### **Given**

ullet image sequence  $f(x_1,x_2,x_3)$ 

| location | $(x_1,x_2)$ | $\in$ | ${f \Omega}$ |
|----------|-------------|-------|--------------|
| time     | $x_3$       | $\in$ | [0,T]        |

#### Wanted

• interframe displacement field  $\mathbf{u} = (u_1, u_2, 1) \rightarrow \mathsf{optic}$  flow



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# Introduction (2)

# What is Optic Flow Good for?

- Extraction of Motion Information
  - navigation
  - obstacle detection
  - tracking
- Processing of Image Sequences
  - compact coding (compression  $\rightarrow$  MPEG)
  - restoration and editing
  - motion compensation
- Related Correspondence Problems
  - stereo reconstruction
  - structure-from-motion
  - medical image registration







# Introduction (3)

# Why Variational Methods?

### Many Advantages

- transparent modelling
- well-posedness and simple minimisation
- highest accuracy in the literature
- dense flow fields

### Main Drawback

large linear/nonlinear systems of equations
 (→ very slow with basic numerical solvers)

# **Goals of this Talk**

- Quality: introduction to the design of high accuracy methods
- Efficiency: discussion of specifically adapted real-time algorithms
- Practical Relevance: presentation of current applications

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# Outline

# Outline

### Modelling

- general structure
- data and smoothness term
- qualitative benchmarks

### Numerics

- minimisation and discretisation
- efficient multigrid algorithms
- performance benchmarks
- real-time live demo

# Applications

### Summary

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Outline

# **PART I** Modelling

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# **Motion Estimation as Optimisation Problem**

 Optic Flow u as Minimiser of the Energy Functional (Horn/Schunck 1981)

$$E(\mathbf{u}) = \int \left(\underbrace{D(\mathbf{u})}_{\mathsf{Data Term}} + \alpha \underbrace{G(\nabla \mathbf{u})}_{\mathsf{Smoothness Term}}\right) d\mathbf{x}$$

- data term penalises deviations from constancy assumptions on image features
- smoothness term penalises deviations from smoothness of solution
- regularisation parameter  $\alpha > 0$  determines smoothness
- global method: integration over single image or full video

# **Standard Data Term**

Constancy Assumption on the Image Brightness (e.g. Horn/Schunck 1981, Lucas/Kanade 1981)

• implicit formulation

$$0 = f(x_1 + u_1, x_2 + u_2, x_3 + 1) - f(x_1, x_2, x_3)$$

• Taylor linearisation

$$0 = f_{x_1} \boldsymbol{u}_1 + f_{x_2} \boldsymbol{u}_2 + f_{x_3} \boldsymbol{1} = \boldsymbol{\mathrm{u}}^\top \nabla_3 f$$

• quadratic penalisation

$$(\mathrm{u}^{ op} 
abla_3 f)^2$$

#### Drawback

• image brightness not invariant under varying illumination

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Modelling - The Data Term (2)

# **Higher Order Constancy Assumptions**

#### • Constancy Assumptions on Image Derivatives

(Uras et al. 1988, Schnörr 1993, Papenberg/Bruhn/Brox/Didas/Weickert IJCV 2006)

| Constancy           | Data Term  | Motion Type                                   |
|---------------------|--|---|
| gradient            | $\sum\limits_{i=1}^2 (\mathbf{u}^	op oldsymbol{ abla}_3 oldsymbol{f_{x_i}})^2$             | translational<br>divergent<br>slow rotational |
| Hessian             | $\sum\limits_{i=1}^2 \sum\limits_{j=1}^2 (\mathbf{u}^	op  abla_3 oldsymbol{f_{x_ix_j}})^2$ | translational<br>divergent<br>slow rotational |
| gradient magnitude  | $(\mathbf{u}^{	op}  abla_3    abla_2 f  )^2$   | any   |
| Hessian trace       | $(\mathrm{u}^{	op}  abla_3({\Delta_2 f}))^2$   | any   |
| Hessian determinant | $(\mathrm{u}^{	op}  abla_3 \mathrm{det}(\mathcal{H}_2 f))^2$                               | any   |

#### Drawback

• images derivatives only invariant under **global additive** illumination changes

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# **Colour Constancy Assumptions**

- Constancy Assumptions on Photometric Invariants (*Mileva/Bruhn/Weickert DAGM 2007*)
  - colour images offer three measurements per pixel (R,G,B)
  - exploit redundancy by computing differences and ratios
  - $\bullet\,$  transformations of the colour space / normalisation of RGB channels

$$(R,G,B)^ op \mapsto \; \left(rac{R}{N},rac{G}{N},rac{B}{N}
ight)^ op, \qquad N=rac{R+G+B}{3}$$

• invariant under more realistic **local multiplicative** illumination changes



DIPLODOC Road Sequence

RGB Constancy

Invariant Constancy

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# **Generic Framework for the Data Term**

Motion Tensor Formalism

(Bigün/Granlund/Wiklund 1991, Bruhn/Weickert/Kohlberger/Schnörr IJCV 2006)

- compact representation for combined data term
- ullet consider n constancy assumptions on  $p_1,...,p_n$  with weights  $\lambda_1,...,\lambda_n$

$$\sum_{i=1}^{n} \lambda_{i} (\mathbf{u}^{\top} \nabla_{3} p_{i})^{2} = \sum_{i=1}^{n} \lambda_{i} (\mathbf{u}^{\top} \nabla_{3} p_{i} \nabla_{3} p_{i}^{\top} \mathbf{u})$$
$$= \mathbf{u}^{\top} \left( \sum_{i=1}^{n} \lambda_{i} \nabla_{3} p_{i} \nabla_{3} p_{i}^{\top} \right) \mathbf{u} = \mathbf{u}^{\top} J \mathbf{u}$$

ullet single quadratic form with positive semi-definite 3  $\times$  3 motion tensor J

#### Advantages

- framework for all presented data terms
- rank analysis specifies degrees of freedom

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Modelling - The Data Term (5)

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### **Generic Framework for the Data Term**

#### • Overview of Motion Tensors

(Bruhn/Weickert/Kohlberger/Schnörr IJCV 2006)

| Constancy           | Motion Tensor J  |
|---------------------|--|
| brightness          | $ abla_3 oldsymbol{f} \  abla_3 oldsymbol{f}^	op$  |
| gradient            | $\sum\limits_{i=1}^2 ( abla_3 oldsymbol{f_{x_i}}) ( abla_3 oldsymbol{f_{x_i}})^	op$  |
| Hessian             | $\sum\limits_{i=1}^2 \sum\limits_{j=1}^2 ( abla_3 oldsymbol{f_{x_ix_j}}) ( abla_3 oldsymbol{f_{x_ix_j}})^	op$  |
| gradient norm       | $\frac{(f_{x_1} \nabla_3 f_{x_1} + f_{x_2} \nabla_3 f_{x_2})(f_{x_1} \nabla_3 f_{x_1} + f_{x_2} \nabla_3 f_{x_2})^\top}{f_{x_1}^2 + f_{x_2}^2}$  |
| Hessian trace       | $(  abla_3 {\displaystyle\sum\limits_{i=1}^2 f_{x_i x_i}}) (  abla_3 {\displaystyle\sum\limits_{i=1}^2 f_{x_i x_i}})^	op$  |
| Hessian determinant | $egin{aligned} &(f_{x_2x_2} abla_3f_{x_1x_1}\!+\!f_{x_1x_1} abla_3f_{x_2x_2}\!-\!2f_{x_1x_2} abla_3f_{x_1x_2})&\cdot\ &(f_{x_2x_2} abla_3f_{x_1x_1}\!+\!f_{x_1x_1} abla_3f_{x_2x_2}\!-\!2f_{x_1x_2} abla_3f_{x_1x_2})^	op \end{aligned}$ |

Modelling - The Data Term (6)

# **Robustification against Noise and Outliers**

• Local Least Squares Fit

(Lucas/Kanade 1981, Bruhn/Weickert/Schnörr IJCV 2005)

• integration over a neighbourhood of fixed size

$$\boldsymbol{K_{\rho}} \ast \left( \mathbf{u}^{\top} \boldsymbol{J} \; \mathbf{u} \right) = \mathbf{u}^{\top} \left( \boldsymbol{K_{\rho}} \ast \boldsymbol{J} \; \right) \, \mathbf{u} = \mathbf{u}^{\top} \boldsymbol{J_{\rho}} \; \mathbf{u}$$

#### Robust Statistics – Single Assumption

(Black/Anandan 1991, Mémin/Pérez 1998)

• subquadratic penalisation with increasing function  $\Psi(s^2)$ 

 $\Psi(\mathbf{u}^{\top}J \mathbf{u})$ 

ullet reduce influence of outliers, e.g. by replacing  $L_2$  with  $L_1$  norm

$$\Psi(s^2):=s^2 \quad o \quad \Psi(s^2) \ := \ \sqrt{arepsilon^2+s^2}-arepsilon$$

Modelling - The Data Term (7)

# **Robustification of Multiple Assumptions**

Robust Statistics – Correlated Assumptions (Brox/Bruhn/Papenberg/Weickert ECCV 2004)

• joint robustification, e.g. in the case of RGB colour images

$$\Psi \, (\, \sum_{i=1}^n \lambda_i \; \mathrm{u}^ op J_i \; \mathrm{u} \,)$$

**Robust Statistics – Independent Assumptions** 

(Bruhn/Weickert ICCV 2005)

• **separate** robustification, e.g. in the case of HSV colour images

$$\sum_{i=1}^n \lambda_i \ \Psi \left( \mathrm{u}^ op J_i \ \mathrm{u} 
ight)$$

### **Large Displacements**

### Theoretically Justified Warping

(Nagel 1983, Brox/Bruhn/Papenberg/Weickert ECCV 2004)

• original constancy assumption

$$0 = f(\mathbf{x} + \mathbf{u}) - f(\mathbf{x})$$

• incremental computation

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \mathbf{\Delta}\mathbf{u}^k$$

• linearisation only by increment

 $0 = \mathbf{\Delta}\mathbf{u}^{k\top} \nabla_3 f(\mathbf{x} + \mathbf{u}^k)$ 

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# Large Displacements

### Multiscale Strategy

(Bergen/Anandan/Hanna/Hingorani 1992)

• large displacements become small displacements



fine scale (displacements up to 10 pixels)



coarse scale (displacements up to 1 pixel)

- Modified Motion Tensors for Large Displacements (Bruhn/Weickert/Kohlberger/Schnörr IJCV 2006)
  - motion tensor notation still applicable (in the incremental computation)
  - specific multiscale representation

# **Spatial vs. Spatiotemporal Regularisation**

- Spatial Regularisation (Horn/Schunck 1981)
  - penalises deviations from smoothness in the spatial domain

$$\sum_{i=1}^2 |
abla_2 u_i|^2$$

### Spatiotemporal Regularisation

(Nagel 1990, Weickert/Schnörr 1999)

• extending spatial smoothness to the temporal domain

$$\sum_{i=1}^{2} | \mathbf{\nabla}_{\mathbf{3}} u_i |^2$$

- computationally hardly more expensive than spatial approach
- better results but delayed computation (stack of frames required)

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# **Adaptive Smoothness Terms**

 Diffusion-Inspired Regularisers (yield Diffusion Tensors) (Weickert/Schnörr 2001, Zimmer/Bruhn at al. EMMCVPR 2009)

| Strategy  | Smoothness Term  |
|---|--|
| homogeneous<br>( <i>Horn/Schunck 1981</i> )   | $\sum_i   oldsymbol{ abla} u_i  ^2$  |
| <b>image-driven</b> , isotropic/anistropic<br>( <i>Alvarez et al. 1999, Nagel 1983</i> )    | $\left  egin{array}{l} g(  abla f ^2) \sum_i   abla u_i ^2 \Big/ \sum_i  abla u_i^	op D( abla f)   abla u_i  ight.$                            |
| <b>flow-driven</b> , isotropic/anisotropic<br>( <i>Schnörr 1994, Weickert et al. 2001</i> ) | $egin{aligned} &\Psi\Big(\sum_i   abla u_i ^2\Big) \ \Big/ \ 	ext{tr} \left(\Psi\Big(\sum_i  abla u_i  abla u_i^	op\Big)  ight) \end{aligned}$ |
| <b>combined</b> , anisotropic<br>( <i>Sun et al. 2008</i> )                                 | $\sum_{r} \Psi_{r} \Big( \sum_{i} \nabla u_{i}^{\top} D_{r} (\nabla f) \nabla u_{i} \Big)$   |
| <b>complementary</b> , anisotropic<br>( <i>Zimmer/Bruhn et al. 2009</i> )                   | not yet available  |

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# **Adaptive Smoothness Terms**

#### Comparison of Different Strategies

(Zimmer/Bruhn/Weickert/Valgaerts/Salgado/Rosenhahn/Seidel EMMCVPR 2009)



# **Real-World Sequences - Qualitative Evaluation**

#### Rheinhafen Sequence

(Nagel, Size  $688 \times 565 \times 1000$ )





Frame 1130

Brightness Constancy Homogeneous Regulariser

# **Real-World Sequences - Qualitative Evaluation**

#### Rheinhafen Sequence

(Nagel, Size  $688 \times 565 \times 1000$ )





Frame 1130

Brightness Constancy Image-Driven Anisotropic Regulariser

# **Real-World Sequences - Qualitative Evaluation**

#### Rheinhafen Sequence

(Nagel, Size  $688 \times 565 \times 1000$ )



Frame 1130



**Robust** Brightness Constancy **Flow-Driven** Isotropic Regulariser

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### **Real-World Sequences - Qualitative Evaluation**

#### Rheinhafen Sequence

(Nagel, Size  $688 \times 565 \times 1000$ )



Frame 1130



Robust Brightness Constancy Robust Gradient Constancy without Linearisation Flow-Driven Isotropic Regulariser

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### **Real-World Sequences - Qualitative Evaluation**

 Karl Wilhelm Street and Ettlinger Tor Sequence (Nagel, Size 351 × 283 × 1034 and Size 512 × 512 × 50)



Karl Wilhelm Street



Ettlinger Tor

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# **Synthetic Sequences - Qualitative Evaluation**





Frame 8

Frame 9

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# **Synthetic Sequences - Qualitative Evaluation**



Ground Truth (Colour Plot)



Ground Truth (Vector Plot)

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**Synthetic Sequences - Qualitative Evaluation** 

 Yosemite Sequence with Clouds (Quam 1984, Size 316 × 252 × 15)





Brightness Constancy Homogeneous Regulariser

AAE=7.17°

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**Synthetic Sequences - Qualitative Evaluation** 





Ground Truth (Colour Plot)

Brightness Constancy Image-Driven, Anisotropic Regulariser

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# **Synthetic Sequences - Qualitative Evaluation**



Ground Truth (Colour Plot)

AAE=5.74°

**Robust** Brightness Constancy **Flow-Driven**, Isotropic Regulariser

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# **Synthetic Sequences - Qualitative Evaluation**



Ground Truth (Colour Plot)

 $AAE=2.42^{\circ}$ 

Robust Brightness Constancy Robust Gradient Constancy without Linearisation Flow-Driven, Isotropic Regulariser



### Synthetic Sequences - Comparison to Literature

#### Yosemite Sequence with Clouds

(Quam 1984, Size  $316 \times 252 \times 15$ )

| Technique             | AAE           |
|-----------------------|---------------|
| Horn/Schunck, orig.   | 31.69°        |
| Singh, step 1         | 15.28°        |
| Anandan               | 13.36°        |
| Singh, step 2         | 10.44°        |
| Nagel                 | 10.22°        |
| Horn/Schunck, mod.    | 9.78°         |
| Uras <i>et al.</i>    | 8.94°         |
| Prototype A           | <b>7.17</b> ° |
| Liu <i>et al.</i>     | 6.85°         |
| Prototype B           | <b>6.44</b> ° |
| Prototype E           | 6.42°         |
| Prototype D           | 6.32°         |
| Prototype C           | <b>6.28</b> ° |
| Prototype F (2-D, SD) | <b>5.74</b> ° |
| Alvarez et al.        | 5.53°         |

| Technique             | AAE           |
|-----------------------|---------------|
| Mémin/Pérez           | 5.38°         |
| Prototype F (3-D, SD) | <b>5.18</b> ° |
| Farnebäck             | 4.84°         |
| Mémin/Pérez           | 4.69°         |
| Prototype F (3-D, LD) | <b>4.17</b> ° |
| Wu <i>et al.</i>      | 3.54°         |
| Prototype G (2-D, SD) | <b>3.50</b> ° |
| Prototype G (3-D, SD) | <b>2.78</b> ° |
| Teng <i>et al.</i>    | 2.70°         |
| Prototype H (2-D, LD) | <b>2.42°</b>  |
| Amiaz/Kiryati         | 2.04°         |
| Prototype G (3-D, LD) | <b>1.78</b> ° |
| Amiaz/Kiryati         | 1.73°         |
| Prototype H (3-D, LD) | <b>1.72</b> ° |
| Brox/Bruhn/Weickert   | <b>1.22</b> ° |

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# **Synthetic Sequences - Comparison to Literature**

#### Middlebury Benchmark

(Baker/Scharstein/Lewis/Roth/Black/Szeliski 2007, 8 Sequences with Different Sizes)

|                         |      |                                 |                                     |  | 5                              |                                |                                |                                |                                |
|-------------------------|------|---------------------------------|-------------------------------------|--|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| Average                 |      | Army                            | Mequon                              | Schefflera                                 | Wooden                         | Grove                          | Urban                          | Yosemite                       | Teddy                          |
| angle                   |      | (Hidden texture)                | (Hidden texture)                    | (Hidden texture)                           | (Hidden texture)               | (Synthetic)                    | (Synthetic)                    | (Synthetic)                    | (Stereo)                       |
| error                   | avg. | <u>GI mu mi</u>                 |                                     | <u>GI IMU IM1</u>                          | <u>GI mu mi</u>                |                                |                                |                                |                                |
|                         | rank | <u>all disc untext</u>          | <u>all disc untext</u>              | <u>all disc untext</u>                     | <u>all disc untext</u>         | <u>all disc untext</u>         | <u>all disc untext</u>         | <u>all disc untext</u>         | <u>all disc untext</u>         |
| CompIOF [27]            | 4.2  | <u>4.44</u> 8 11.27 4.048       | 2.51 2 9.77 3 1.74 1                | 3.93 + 10.6 + 2.04 +                       | <u>3.87</u> 7 18.8↓ 2.196      | 3.17 1 4.00 1 2.92 2           | <u>4.64</u> 5 13.82 3.644      | 2.17 6 3.36 3 2.51 12          | <u>3.08</u> 2 7.042 3.655      |
| Adaptive [26]           | 4.3  | 3.29 1 9.43 1 2.28 1            | <u>3.10</u> 6 11.47 2.467           | 6.589 15.78 2.525                          | 3.14 1 15.6 1 1.56 1           | <u>3.67</u> 6 4.465 3.486      | 3.32 1 13.0 1 2.38 1           | <u>2.76</u> 11 4.39 10 1.93 7  | <u>3.58</u> 3 8.18 3 2.88 2    |
| Spatially variant [22]  | 5.9  | <u>3.73</u> 3 10.25 3.33↓       | <u>3.02</u> 5 11.06 2.678           | <u>5.36</u> 5 13.86 2.352                  | <u>3.67</u> 2 19.36 1.843      | <u>3.81</u> 8 4.81 13 3.69 9   | 4.48 4 16.07 3.90 5            | 2.11 4 3.26 2 2.12 9           | <u>4.66</u> 8 9.41 8 4.35 10   |
| TV-L1-improved [20]     | 6.8  | 3.36 2 9.63 2 2.62 2            | 2.82 4 10.7 5 2.23 3                | 6.50 8 15.8 9 2.73 6                       | <u>3.80</u> 5 21.3 11 1.76 2   | 3.34 2 4.38 4 2.39 1           | <u>5.97</u> 7 18.1 13 5.67 11  | <u>3.57</u> 15 4.92 16 3.43 17 | <u>4.01</u> 7 9.849 3.443      |
| Multicue MRF [24]       | 7.9  | <u>4.50</u> 9 10.1 3 4.18 12    | 2.52 3 7.07 1 2.36 6                | 3.09 1 7.41 1 2.36 3                       | <u>4.46</u> 10 20.8 10 2.73 9  | 3.51 4 4.11 2 4.06 14          | 6.089 15.66 5.409              | 5.25 22 5.36 18 9.02 22        | 3.63 + 8.39 + 4.15 8           |
| F-TV-L1 [18]            | 8.5  | <u>5.44</u> 1212.5 11 5.69 16   | <u>5.46</u> 13 15.0 13 4.03 13      | 7.48 13 16.3 11 3.42 11                    | 5.08 12 23.3 15 2.81 10        | <u>3.42</u> 3 4.343 3.033      | 4.05 3 15.1 4 3.18 2           | 2.43 8 3.92 8 1.87 6           | 3.90 6 9.357 2.61 1            |
| DPOF [21]               | 10.2 | <u>5.63</u> 13 10.9 6 4.16 11   | <u>4.05</u> 10 12.1 8 3.31 9        | <u>3.87</u> 3 8.822 3.179                  | 4.34 8 16.2 2 3.13 12          | <u>3.95</u> 12 4.78 12 4.17 16 | 6.69 15 15.2 5 6.27 14         | 5.62 23 6.89 24 6.60 21        | 2.44 1 4.83 1 3.74 7           |
| Brox et al. [8]         | 10.2 | <u>4.80</u> 11 14.4 15 4.29 13  | <u>4.05</u> 10 13.5 10 3.71 11      | <u>6.63</u> 10 16.0 10 7.26 13             | 5.22 13 22.7 14 3.22 13        | 4.56 17 6.09 23 3.40 4         | 3.97 2 17.9 11 3.41 3          | 2.07 3 3.766 1.182             | <u>5.14</u> 10 11.9 12 4.28 9  |
| Fusion (9)              | 10.7 | <u>4.43</u> 7 13.7 13 4.08 9    | 2.47 1 8.91 2 2.24 4                | <u>3.70</u> 2 9.683 3.128                  | 3.68 3 19.87 2.54 8            | <u>4.26</u> 15 5.16 14 4.31 18 | 6.32 11 16.8 9 6.15 13         | <u>4.55</u> 19 5.78 🗙 3.10 16  | 7.12 18 13.6 18 7.86 19        |
| SegOF [13]              | 11.2 | <u>5.85</u> 14 13.5 12 3.98 7   | 7.40 15 14.9 12 8.13 19             | 8.55 15 17.3 15 9.01 14                    | <u>6.50</u> 16 18.1 3 5.14 16  | <u>3.90</u> 11 4.53 6 4.81 21  | 6.57 14 21.7 19 6.81 17        | 1.65 1 3.49 5 1.08 1           | <u>3.71</u> 5 9.23 6 3.63 4    |
| Dynamic MRF [10]        | 11.2 | <u>4.58</u> 10 12.4 10 4.14 10  | <u>3.25</u> 8 13.9 11 2.27 5        | <u>6.02</u> 7 16.8 <u>12</u> 2.36 <u>3</u> | <u>4.39</u> 9 22.6 13 2.51 7   | <u>3.61</u> 5 4.557 3.46 5     | <u>6.81</u> 16 22.2 21 6.78 16 | 2.41 7 3.48 4 3.69 18          | 9.26 22 17.8 22 10.2 22        |
| CBF [15]                | 12.1 | <u>3.95</u> 5 10.1 3 3.44 6     | <u>3.70</u> 9 10.6 <b>4</b> 3.85 t2 | <u>5.64</u> 6 13.55 3.34 to                | <u>3.71</u> 4 21.5 12 1.99 4   | <u>4.36</u> 16 5.50 16 3.55 7  | <u>11.3</u> 23 19.1 15 9.05 22 | 6.79 25 7.37 26 11.6 25        | <u>5.50</u> 11 11.8 11 5.66 13 |
| GraphCuts [17]          | 13.2 | <u>6.25</u> 15 1 4.3 14 5.53 15 | 8.60 17 20.1 19 6.61 15             | 7.91 14 15.47 10.9 15                      | <u>4.88</u> 11 19.0 5 3.05 11  | <u>3.78</u> 7 4.71 to 3.94 to  | 8.74 19 16.4 8 5.39 8          | 4.04 18 4.87 14 4.85 20        | 6.35 14 12.2 13 6.05 16        |
| Learning Flow [14]      | 13.2 | 4.23 6 11.7 9 3.41 5            | 4.16 12 15.3 14 3.42 10             | 6.78 11 16.9 13 3.83 12                    | <u>6.41</u> 15 25.3 17 4.25 14 | 4.66 20 6.01 22 4.00 13        | 6.33 13 20.7 17 5.30 6         | <u>3.09</u> 13 4.84 13 2.91 14 | 7.08 17 15.0 20 5.27 12        |
| Second-order prior [11] | 13.5 | <u>3.84</u> ↓ 11.27 3.11 3      | <u>3.12</u> 7 12.99 2.17 2          | <u>6.96</u> 12 17.2 14 2.83 7              | <u>3.84</u> 6 20.59 2.095      | 4.83 22 5.83 20 3.90 11        | 14.0 25 21.8 20 8.28 19        | 7.74 25 6.88 23 11.7 25        | <u>6.74</u> 16 13.4 17 5.80 14 |

Outline

# **PART II** Numerics

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# **Minimisation Strategy**

### • Euler–Lagrange Equations

- necessary conditions for a minimiser
- coupled system of partial differential equations
- discretisation yields large linear or nonlinear system of equations
- typically solved by iterative methods (Jacobi, Gauß-Seidel)

### Drawback of Iterative Methods

- slow convergence after a few iterations
- logarithmic error spectrum reveals slow decrease of lower frequency parts (→ only efficient damping of higher error frequency parts)



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# **Multigrid Methods**

Basic Idea

(Brandt 1977, Hackbusch 1985)

- transfer and compute error on coarser grids
- low frequencies reappear as higher frequencies
   (→ also efficient damping of lower error frequency parts)

### Recursive Strategies, Linear Complexity

- hierarchical application ( $\rightarrow$  V–cycle, W–cycle)
- additional usage of hierarchical initialisation ( $\rightarrow$  Full Multigrid)



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Basic Idea

(Brandt 1977, Hackbusch 1985)

- transfer and compute error on coarser grids
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# **Multigrid for Optic Flow**

#### Specific Adaptations

- improved coarse grid correction scheme (tensor based) (Bruhn/Weickert/Kohlberger/Schnörr IJCV 2006)
- improved intergrid transfer operators (non-dyadic) and solvers (coupled) (Bruhn/Feddern/Weickert/Kohlberger/Schnörr IEEE TIP 2005)
- extended to large displacements (combination with warping) (Bruhn/Weickert ICCV 2005)

#### Overview of Multigrid Implementations

| Туре                                       | MG Solver      | Cycles | Basic Solver | Pre/Post |
|--|----------------|--------|--------------|----------|
| A - Homogeneous                            | FMG-W          | 1      | GS-CPR       | 1-1      |
| <b>B</b> - Image-Driven Isotropic          | FMG-W          | 2      | GS-CPR       | 2-2      |
| C - Image-Driven Anisotropic               | FMG-W          | 4      | GS-ALR       | 1-1      |
| <b>D</b> - Flow-Driven Isotropic           | FAS-FMG-W      | 2      | GS-CPR       | 2-2      |
| <b>E</b> - Flow-Driven Anisotropic         | FAS-FMG-W      | 4      | GS-ALR       | 1-1      |
| <b>F</b> - Bruhn <i>et al.</i> 2-D, SD     | FAS-FMG-W      | 2      | GS-CPR       | 2-2      |
| <b>G</b> - Papenberg <i>et al.</i> 3-D, SD | FAS-FMG-W      | 2      | GS-CPR       | 2-2      |
| H - Bruhn/Weickert 2-D, LD                 | WARP-FAS-FMG-W | 2      | GS-CPR       | 3-3      |

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# Comparison of Numerical Solvers (Image Size $160 \times 120$ )

#### • Overview for Different Numerical Prototypes

- C implementation on standard desktop PC (3.06 GHz Pentium4)
- $\bullet\,$  stopping criterion: norm of error less than 1% of norm of solution

#### Method with Image-Driven Anisotropic Regularisation: Linear Multigrid

| Solver               | Iterations | Time [s] | FPS $[s^{-1}]$ | Speedup |
|----------------------|------------|----------|----------------|---------|
| Mod. Explicit Scheme | 36433      | 47.08    | 0.02           | 1       |
| Gauß-Seidel (ALR)    | 607        | 3.60     | 0.27           | 13      |
| Full Multigrid       | 1          | 0.17     | 5.88           | 275     |

#### Method with Flow-Driven Isotropic Regularisation: Nonlinear Multigrid

| Solver               | Iterations | Time [s] | FPS $[s^{-1}]$ | Speedup |
|----------------------|------------|----------|----------------|---------|
| Mod. Explicit Scheme | 10633      | 30.492   | 0.033          | 1       |
| Gauß-Seidel (ALR)    | 2679       | 6.911    | 0.145          | 4       |
| FAS Full Multigrid   | 1          | 0.082    | 12.172         | 372     |

**Experiments - Performance Benchmarks (2)** 

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# Multigrid Speedups (Image Size $160 \times 120$ )

• Overview For Different Model Prototypes (Bruhn/Weickert/Kohlberger/Schnörr IJCV 2006)

- two to three orders of magnitude for different regularisation strategies

| Туре                               | Solver             | FPS  | Speedup |
|------------------------------------|--------------------|------|---------|
| A - Homogeneous                    | Full Multigrid     | 62.7 | 220     |
| <b>B</b> - Image-Driven Isotropic  | Full Multigrid     | 20.8 | 251     |
| C - Image-Driven Anisotropic       | Full Multigrid     | 5.8  | 275     |
| D - Flow-Driven Isotropic          | FAS Full Multigrid | 12.1 | 372     |
| <b>E</b> - Flow-Driven Anisotropic | FAS Full Multigrid | 2.0  | 120     |

• three to four orders of magnitude for high accuracy methods

| Туре                                       | Solver                  | FPS  | Speedup |
|--|-------------------------|------|---------|
| <b>F</b> - Bruhn <i>et al.</i> 2-D, SD     | FAS Full Multigrid      | 11.5 | 2836    |
| <b>G</b> - Papenberg <i>et al.</i> 3-D, SD | FAS Full Multigrid      | 9.9  | 10588   |
| <b>H</b> - Bruhn/Weickert 2-D, LD          | Warp FAS Full Multigrid | 2.9  | 5454    |

**Experiments - Performance Benchmarks (3)** 

# Speedup by Parallel Hardware (Image Size 316 imes 252)

Cell Processor - Sony Playstation 3

(Gwosdek/Bruhn/Weickert VMV 2008, Gwosdek/Bruhn/Weickert JRTIP 2009 submitted)

- 6 SPUs with ringbus memory interface
- Speedup of 6.5 compared to a 3.2 GHz desktop PC
- linear case: up to 210 dense flow fields per second (13.6 million pixels)
- **nonlinear case:** up to 65 dense flow fields per second (4.2 million pixels)





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#### **Real-Time Live Demo**

• Live Computation with Webcam  $(160 \times 120)$ 



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Outline

# **PART III** Applications

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Applications - Stereo (1)

#### **Stereo - Calibrated Case**

- Integration of Prior Knowledge on the Stereo Geometry (Slesareva/Bruhn/Weickert DAGM 2005)
  - restriction of search space to given stereo geometry
  - depth can be directly computed from displacements (disparity)
- **Example: Reconstruction of the Pentagon from Aerial Views** (*CMU Stereo Database, Size* 512 × 512)



CMU Pentagon Image Pair with Stereo Geometry

**Displacement Field** 

Reconstruction with Illumination

**Applications - Stereo (2)** 

#### **Stereo - Uncalibrated Case**

- Joint Estimation of Displacements and Stereo Geometry (Valgaerts/Bruhn/Weickert DAGM 2008)
  - more precise and more robust estimation of correspondences
  - more exact estimation of camera poses (essential matrix)
- Example: Face Reconstruction from Uncalibrated Images (*Pascal Gwosdek, Size 280* × 430)



Image Pair without Geometry

Reconstruction with Texture

# Deinterlacing

- Conversion from Interlaced to Progressive Format (Ghodstinat/Bruhn/Weickert SGAVMA 2009)
  - alternatingly only even and odd lines given (z.B. PAL)
  - inpainting of missing information respecting motion trajectories
- **Example: Motion Compensation in Broadcasts of Sports Events** (*European Broadcasting Union, Zoom-In, Size 300* × *300*)



Progressive Image

Interlaced Image

Deinterlaced Result

Displacement Field

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# **Hairstyle Simulation**

- Automatic Registration of Reference Hairstyles onto Customer Faces (Demetz/Weickert/Bruhn/Welk SSVM 2007)
  - adaptation of reference hairstyle according to deformation field
  - preregistration of eyes, masking of hairstyle, flow computation

**Example: Registration of a Short Hairstyle** (*Style Concept, Size 900* × *900*)



Customer Face F



Reference Face with Hairstyle



Hairstyle Simulation

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# **Motion Analysis**

- Motion Estimation of Nanoparticles in Tracheal Tissue (Kariger/Bruhn/Henning/Weickert/Lehr - Cooperation with Dept. of Pharmaceutical Technology)
  - investigation of mucociliary clearance (protection of respiratory system)
  - smoothness constraints from the field of particle image velocimetry (PIV)
- Example: Transport of Coal Particles of Size 1-100  $\mu$ m (Andreas Henning, Size 660  $\times$  492)



Images with Time Interval 200 ms

Registered Images Velocity 2.55 mm/min

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### Summary

# Summary

#### High Accuracy Models

- generic framework for the design of novel methods
- highest precision in the literature
- robust under noise and illumination changes

#### • Real-Time Algorithms

- speedups of two to four orders of magnitude
- additional acceleration using parallel hardware

#### Numerous Applications

- stereo: calibrated and uncalibrated case
- video processing: deinterlacing, re-timing
- image registration: hairstyle simulation, particle matching

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# Thank you very much!

# more information: www.mia.uni-saarland.de

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|            |              |             |              |             |                     |             |            |               |   |            |   |              | -   | -                |   |  |  | -              |          |           |              |             |  |  |                   |
|            | •            |             |              | -           | -                   | -           | -          |               |   |            |   |              |   |                  | -   | -  | -  | -              |          |           |              |             |  |  | -                 |
| •          | •            | -           |              | ٠           |                     |             |            |               |   | •          | •   | •            | -   | -                | -   |  | -  | -              |          |           |              |             |  |  |                   |
|            |              |             | а.           |             | *                   | +           | -          | ų.            | 4   |            |   |              | ня.   | -                | -   |  |  | -              |          | 14        | -            | -           |  |  |                   |
| •          | ٠            | ٠           | ٠            | ٠           | +                   | +           | +          | +             | +   | -          | +   | ар.          | 19 <b>1</b> -   | -+               | -+  |  | -+   |                | -        | -         | -            |             |  |  |                   |
| •          | ٠            | -           | *            | +           | +                   | +           | 4          | +             | 4   | -          | н.  | -            | -   | -+               | 8- <b>4</b> -   |  |  | 10- <b>1</b> - |          | -         | -            | **          | Sarage Sarage Sarage Strengt   |  | ** ** ***         |
| +          | ٠            | ÷           | ٠            | ÷           | -                   | -           | +          | +             | +   |            | -   | -            | 14 <u>0</u>   |                  |   |  |  |                |          | **        | -+           | *           |  |  | ** ** ***         |
| ٠          | •            | -           | ٠            | ٠           | ٠                   | ٠           | •          | •             | •   | •          | 14  | **           | 5 <b>4</b>  | in g             | in the second | in a g   | in a   | t-a            | ••       | ••        | -            |             | Sand and the state of the state |  |                   |
| •          | -            | -           | 14           | 4           | 4                   |             | -          | -             | -   | -          | -   | -            | -   | 1- <b>4</b>      | trage.  | i-a  | ing.   | terige.        |          | -         | ing.         |             |  |  | hang kang kang    |
| ~          | -            | **          | **           | **          | **                  | **          | -          | 1             | -   | **         | -   | -            | -   | -                | ing.  | 1-192.   | 1  | 1-12.          | -        |           | -            |             | tering tering tering tering ter  | and a second provide the second s   |                   |
| -          |              | -           | 14           | -           | -1- <del>1</del> 0. | - trips     | 14         | 1.            | 1.4   | -          | ting.   | ting.        | train.  | 1. AN            | ting.   | tinge.   | ing.   | ing.           | ting.    |           | ing.         | 1- <b>6</b> | Surgering, Surgering, Surgering, Surgering, Sur  |  | hog hog hog       |
| -          | -            | 10          | 16           | 14 <u>6</u> | -1-1 <u>0</u> -     | - 1964<br>1 | 1          | 1             | -   | 1.00       | traja.  | 1-1 <b>2</b> | 1-96.   | 1-1 <b>9</b> -   |   | 100 <b>10</b> 0  | 1-1 <b>-1</b> -1   | -              | •        | •         |              | -           | Contention, Contention, Contention, Southern   |  | ing ing ing       |
| °в.        | 1            | 100         | ÷ц.          | ~           | 1                   | 14 <u>1</u> | άης.       | ά <b>η</b> ς. | ά <b>ης</b> ,   | 1          | ing.  | ing.         | 1-12.   | 1-1 <u>+</u>     | 1-1-1-1<br>   |  | ing a  | ing.           | •        | •         | 19.          | 14          |  | inca <sup>inc</sup> ina <sup>inc</sup> ina i   | ingg ingg ingg    |
| ÷1.        | ι.<br>Έλλ    | ÷           | ά <b>η</b> . | ~           | ÷                   | 1.<br>1.    | ing.       | ing.          | ing.  | ing.       | άφ.   | inge.        | 1-12-   | 1-1 <u>-1</u> -1 | 1. A.   |  | 1-19<br>1  | •              | *        |           | 19 <b>1.</b> | 4           |  | inter internet   | ing ing ing       |
| та,        | 1            | 1990)<br>1  | та,          | 1996.<br>1  | - 1996.<br>1        | 1           | ÷.         | ÷~            | ÷~  | ~ <b>e</b> | 1998 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - 1999 - |              | -1-18 <u>6</u> -  |                  | ~~ <b>.</b>   | - Anna Anna Anna Anna Anna Anna Anna Ann   | in the second se | *              |          | **        | °њ.          | та.         | and the second sec   |  |                   |
| ~          | *            | 19 <b>9</b> | ÷.           | **          | **                  | ÷.,         | 1          | ÷             | ÷.  | ~~         | 1996)<br>1  | 1. A.        | the second  | 1998)<br>1       | 10.00   | the state of the s | ing and  | •              | *        | •         | ά,           | ۰.          |  |  |                   |
| <u>```</u> | ÷.           | 1996 - B.   | ÷е.          | ~           | ~                   | с.          | стан.<br>С | ÷.            | ÷~.   | ~          | 1998<br>1997  | ~~           | the second se | 1995 B.          | 1998<br>1997  | 1999 B   | ingen ander  | 4              | *        | 2         | 2            | *           |  |  | and the second    |
| ~          | ÷.           | 1990.<br>1  | ×.           | **          | 1996.<br>1          | 1998.<br>1  | ÷.         | та.           | 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - 1946 - | 1999 B     | ÷.  | 100.000.     | ingen.  | 199 <sub>8</sub> | -   | - Services   | ing.   | •              | 1        | •         | 2            | *           |  |  | teres teres teres |
| ~          | с <b>н</b> . | стан.<br>1  | та.<br>С     | стан.<br>С  | тан.<br>С           | 1998 - C.   |            | стан.<br>С    |   |            | 1999.<br>1  |              | 1999 <b>- 1</b> 99  | 1998.<br>1       |   | and the second s | 1.00 M   | 1              | ×.       | 2         | 2            | *           |  |  | ~ ~ ~             |
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| 2          | 2            | 2           | 2            |             | <u>`</u>            | <u>`</u>    | <u>`</u>   | 2             | <u>``</u>   | 2          | <u>`</u>  | <u>``</u>    |   | 2                | 2   |  | <u> </u>   | 2              | <u>`</u> | 2         | 2            | È.          |  |  |                   |
| <u>``</u>  | <u>``</u>    | <u>~</u>    | Ì.           | ~           | ~                   |             | ~          | 2             | <u>``</u>   | ~          | 2   | ~            |   | 2                | ~   |  | ~  | ~              | 2        | 2         | 2            | 2           |  |  |                   |
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|--------|-----------|------------|------------|--------|-------|---------------------|---------|--------------|----------|---------|-----|-----|-----|-----|----------|------------|----|-----|------|-----|-------|--------------|--------------|------------|-----|--------------|-------|------------|-----------------|--------------|---------------------------|------------|---------------|
|        |           | -          |            |        |       |                     | •       | •            | • =      |         |     | • = | •   | • = |          |            |    |     |      | -   | -     | -            |              |            |     | •            |       | -          |                 | -            |                           | -          | -             |
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|        | _         | -          |            |        |       |                     | •       |              |          |         |     |     |     |     |          |            |    |     |      |     |       |              |              |            |     |              |       |            |                 | -            |                           |            |               |
|        |           | -          |            |        |       | •                   | •       | •            | •        |         |     | •   | •   | •   |          |            |    |     |      |     | -     | -            |              |            |     |              |       | -          |                 | -            |                           |            | •             |
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|        |           |            | -          |        | -     |                     | •       | •            | •        |         |     | •   | •   | •   |          |            |    |     |      |     |       | -            |              |            |     |              |       | -          |                 | -            |                           |            | •             |
|        |           |            |            |        |       |                     |         |              |          | а.      | н   | а.  |     | •   |          | <b>.</b>   |    |     |      |     |       |              |              |            |     |              |       |            |                 |              | te i                      | in i       | èn i          |
|        |           |            |            | 41 - A |       | -                   |         | -            | ан.<br>Г | а.      | а.  | а.  | •   |     |          | ж.         |    |     |      |     | e - 1 | e - 1        |              |            |     |              | ۰.    | <b>.</b>   | 11 - A          | <b>4</b> - 1 | •                         |            | •             |
|        |           | - 44 A     |            | ÷ - 4  | - 40  | -                   |         | -            |          | а.      | а.  | а.  | а.  | ж.  | <b>.</b> | <b>a</b> 1 |    | а.  | ÷.   |     |       | e - 1        | •            | e - 4      | •   | •            | ۴.    |            | 4               | •            | ÷                         | <b>.</b> . | λ.            |
|        | -         | يو المروال |            | e 1.   | e -e  | -                   | +       | •            | 4        | ж.      | а.  | а,  | а.  | ж.  | ai -     | ai -       |    | н.  | ai - | a   | a     | a - 1        | ė .          | * *        | E I | • •          | a - 1 | 9 - I      | •               | ÷            | 6 - A                     | њ          | а.            |
| +      |           |            |            | er e   | a an  | $\mathcal{L}^{(n)}$ | +       | ÷            | •        | ÷.      | •   | а,  | а.  |     | ж.       | а.         | а. | 4   | ai - | 4   | 4     | e e          | • •          | • •        | •   | r 1          |       | •          | •               | •            | a                         | а.         | 4             |
| de la  | - georgeo | بولجوا     | وشيروش     | er a   | e en  | $e^{-i}$            | er i    | e* 1         | e* -     | •       | •   | ÷.  | ж.  | ж.  | 4        | ÷.         | 4  | 4   | 4    | 4   | 8     | 8 - I        | • •          | • •        | • • |              | 8 - I | •          | •               | s            | 4                         | 4          | н.            |
| æ      | مراسره    | مر شهرت    | والمرجورات | er ar  |       | e.                  | e.      | <i>e</i> * - | e.       | e i     | •   | ٠   | ۰.  | ж.  | а.       | 4          | 4  | а.  | 4    | а.  | 8     | 8 - S        | • •          | • •        |     |              | 8 - I | <b>8</b> - | <b>8</b> - 1    | 4            | 4                         | а,         | а,            |
| æ      | موسروس    | موسيور     | وشريوش     | er er  | a an  | e.                  | er i    | e* -         | e.       | e.      | e i | ÷., |     | ٠   | ж.       | ж.         | ж. | а.  | ۰.   | ÷., | èн –  | 8 - S        | •            |            |     |              | 1     | 10 - I     | ۰.              | ۹.           | ч.                        | ۰.         | •             |
| æ      | مواسروس   | موسمو ا    | ومسحدهم    | e e    | e er  | e.                  | e,      | e.           | e.       | e.      | e.  | e.  | ۶.  |     | ٠        | ۰.         | ۰. | ۰.  | ۰.   | ٠   | ۶.,   | н I          | •            |            |     |              |       | ч.         | ۰.              | ۹            | ۰.                        | •          | ۰.            |
| a.     | مدسر      | ر سر       | ومرم       | e se   | e se  | d.                  | ć.      | e.           | e.       | e.      | ė.  | e.  | 8   | ¢., | а.       | а.         | ۰. | ۰.  | ۴.,  | ۶., | ۴.,   | е - <u>Р</u> | e - 1        | <u>, 1</u> |     |              | ų.    | •          | ۰.              | ч.           | •                         | •          | 8             |
| e.     | مدسري     | ر سر       | بالمراح    | 11     | e de  | e.                  | e.      | e.           | ¢.       | ć.      | ¢.  | ¢.  | 6   | 6   | 6        | а.         | ۰. | £., | Ρ.,  | ۳., | ۳.,   | P (          |              |            |     |              |       | ٩.,        | ۹               | ٩            | ۰.                        | •          | 8             |
| a.     | 11        | ر سر       | ر مر م     | 11     | 1     | ¢.                  | e.      | 1.           | ¢.       | 1.      | 6   | ¢.  | 1   | ۶.  | 6        | ۶.         | ۰. | ۰.  | ۰.   | ۰.  | ۳.,   | • •          | •            |            |     | •            | 1     | ۰.         | а.              | ۰.           | х.                        | а,         | ч.            |
| s.     | 11        | ر زر ا     | ومرمر      | 11     | 1     | 1                   | 1.      | 1.           | 1        | 1.      | ŧ.  | ¢.  | 1   | ۶.  | ۴.       | 6          | ۶. | ۶., | ۳.,  | ۳., | ۳.    | •            |              |            |     |              | ٩.    | ۰.         | ۰.              | ь.           | ч.                        | ч.         | х.            |
| j,     | 11        | 11         | 1          | 11     | 1     | 1                   | 1       | 1.           | 1        | ζ.,     | 1   | ŧ.  | 1   | é.  | é.       | 6          | ۶. | ۴.  | ۴.,  | ۳., | ۳.,   |              |              |            |     | 1            | ۳.,   | ۰.         | х.              | ч.           | ч.                        | ж.         | х.            |
| 2      | رتريم     | ربريخ      | 11         | 11     | 1     | 1                   | 1.      | 1            | 1        | 1.      | 1   | 1   | 6.  | 6.  | 6.       | £.,        | ۶. | ۴., | ۴.,  | •   |       | •            |              | • •        |     | •            | ٩.,   | ۰.         | х.              | Ь.           | Ъ.,                       | х.         | Ъ.            |
| j.     | رتريم     | ر کر م     | 11         | 11     | 1     | 7                   | Ζ.      | 1.           | 1        | 1.      | 1   | Ł   | 6.  | 1   | λ.       | ÷.         | ŧ. | ÷.  | ۴.   | ۴., | Ε.    | 1            | • •          | ۲. I       |     |              | ξ.,   | Ъ.         | Υ.              | Ъ.,          | Ъ.                        | Ъ.         | Ъ.            |
| Ĵ,     | رتريم     | ~/)        | 1          | 11     | 11    | 7                   | $Z_{i}$ | 7            | Ζ.       | ι.      | ŧ.  | £.  | 6.  | Æ.  | é.       | đ.         | ÷. | ŧ.  | ŧ.   | ÷   |       | 8.1          | ŧ 3          | 8 B        |     | <b>k</b> - 1 | λ.    | Ъ.,        | Ъ.,             | Ъ.           | Ъ.                        | Υ.         | λ.            |
| 5      | アメ        | 1          | 1          | 17     | 11    | 7                   | Z       | 71           | Ζ.       | Z       | 7   | Ζ.  | 4.  | 4.  | £.       | £.         | £. | ÷.  | ŧ.,  | ÷., |       | ŧ. i         | ŧ - 3        | 6 3        |     | 5            | λ.    | Ъ.,        | λ.              | Ъ.           | $\mathbf{b}_{1}$          | Ν.         | $\mathcal{M}$ |
| ٢,     | 11        | //,        | 11         | 1      | 11    | 7.                  | Z       | Ζ.           | 7.       | $Z_{i}$ | 7.  | 7.  | Z.  | 4.  | Į.       | £.         | £. | ŧ.  | ŧ.   |     |       | \$ - S       | <b>i</b> - i | k 1        |     | Υ.           | k.    | λ.         | λ.              | λ.           | λ.                        | λ.         | λ.            |
| 8<br>1 | 17,       | 1          | 1          | 11     | 17    | 7                   | 71      | 71           | 7        | $Z^{+}$ | 7   | Ϊ.  | Ì.  | Į.  | Į.       | Į.         | L. | £   | ŧ.   | ł.  | Į.,   | ¥ . 3        | 1            | k 1        | k 1 | k i          | λ.    | ٨.         | $\lambda_{i}$ . | λ.           | $\mathbf{N}_{\mathbf{r}}$ | Χ.         | λ.            |
| ٢,     | 17,       | 11,        | مر م       | 11     | 17    | 7                   | 71      | 7            | 7        | 71      | 7   | 7.  | 7.  | Ì.  | Ì.       | Į.         | L. | 4   | Ł.   | ι.  | ¥     | 1.1          | 1            | L 1        | Ľ   | ١Ľ           | γ.    | ¥.         | X.              | λ.           | $\mathbf{N}$              | Ň.         | X.            |
| 1      | 11,       | 11,        | مر م       | 11     | 14    | 7                   | 71      | 7            | 7        | 71      | 7.  | 7.  | 7.  | Ì.  | Ì.       | Ì.         | Ì. | È.  | 1    | į., | i -   | 11           | í '          | í i        | ( T | Ċ            | Ŷ.    | Ň.         | Ň.              | Ń,           | Ň.                        | Ň.         | Ň.            |
| 1      | 18        | 6 6        | 1.1        | டலி    | 1 A 1 | 18 C. C.            | 6 I I   | e .          | P        | 11 J    |     | F   | e   | Τ., | Г.,      |            |    |     |      |     |       |              | -            |            |     | -            |       |            |                 |              |                           |            |               |

































| Average<br>angle<br>error | avg. | (Hid<br><u>GT</u>    | Army<br>den te<br><u>im0</u> | /<br>xture)<br>i <u>m1</u> | (Ніа<br>(Ніа<br><u>GT</u> | <b>Mequa</b><br>Iden te<br><u>im0</u> | o <b>n</b><br>xture)<br>i <u>m1</u> | So<br>(Hid<br><u>GT</u> | t <b>heffle</b><br>den tex<br>i <u>m0</u> | e <b>ra</b><br>«ture)<br>i <u>m1</u> | <b>V</b><br>(Hid<br><u>GT</u> | Voode<br>dente><br><u>im0</u> | <b>n</b><br>dure)<br>i <u>m1</u> | (S<br><u>GT</u>      | Grove<br>Synthet | ;<br>ic)<br><u>im1</u> | 3)<br><u>GT</u> | Urban<br>Synthet<br><u>im0</u> | i<br>ic)<br>i <u>m1</u>             | ץ (פ<br>פו<br><u>פו</u> | osemi<br>Synthet<br><u>im0</u> | ite<br>ic)<br>i <u>m1</u> | GT                   | Teddy<br>(Stereo<br><u>im0</u> | /<br>))<br>i <u>m1</u> |
|---------------------------|------|----------------------|------------------------------|----------------------------|---------------------------|---------------------------------------|-------------------------------------|-------------------------|---|--------------------------------------|-------------------------------|-------------------------------|----------------------------------|----------------------|------------------|------------------------|-----------------|--------------------------------|-------------------------------------|-------------------------|--------------------------------|---------------------------|----------------------|--------------------------------|------------------------|
|                           | rank | <u>all</u>           | <u>disc</u>                  | <u>untext</u>              | <u>all</u>                | <u>disc</u>                           | <u>untext</u>                       | <u>all</u>              | <u>disc</u>                               | <u>untext</u>                        | <u>all</u>                    | <u>disc</u>                   | <u>untext</u>                    | <u>all</u>           | <u>disc</u>      | <u>untext</u>          | <u>all</u>      | <u>disc</u>                    | <u>untext</u>                       | <u>all</u>              | <u>disc</u>                    | <u>untext</u>             | <u>all</u>           | <u>disc</u>                    | <u>untext</u>          |
| CompIOF [27]              | 4.2  | <u>4.44</u> 8        | 11.21                        | 4.04 8                     | <u>2.51</u> 2             | 9.77                                  | 3 1.74 1                            | <u>3.93</u> <b>i</b>    | 10.6                                      | 2.04 1                               | <u>3.87</u> 1                 | 18.8 <mark>.</mark>           | 2.196                            | <u>3.17</u> 1        | 4.00 1           | 2.92 <mark>2</mark>    | <u>4.64</u> 5   | 13.8 <mark>2</mark>            | 3.64 🕻                              | <u>2.17</u> 6           | 3.36 <mark>3</mark>            | 2.51 12                   | <u>3.08</u> 2        | 7.04 2                         | 3.65 5                 |
| Adaptive [26]             | 4.3  | <u>3.29</u> 1        | 9.43                         | 2.28 1                     | <u>3.10</u> 6             | 11.4                                  | 2.461                               | <u>6.58</u> 9           | 15.78                                     | 2.52 <mark>5</mark>                  | <u>3.14</u> 1                 | 15.6 1                        | 1.56 1                           | <u>3.67</u> 6        | 4.46 5           | 3.48 <mark>6</mark>    | <u>3.32</u> 1   | 13.0 1                         | 2.38 1                              | <u>2.76</u> 11          | 4.39 🛙                         | 1.931                     | <u>3.58</u> 3        | 8.18 <mark>3</mark>            | 2.88                   |
| Spatially variant [22]    | 5.9  | <u>3.73</u> 3        | 10.2                         | 5 3.33 <b>i</b>            | <u>3.02</u> 5             | 11.0                                  | 2.67 8                              | <u>5.36</u> 5           | 13.86                                     | 2.35 <mark>2</mark>                  | <u>3.67</u> 2                 | 19.3 <mark>6</mark>           | 1.84 <mark>3</mark>              | <u>3.81</u> 8        | 4.81 13          | 3.69 9                 | <u>4.48</u> •   | 16.07                          | 3.90 <mark>5</mark>                 | <u>2.11</u> •           | 3.26 <mark>2</mark>            | 2.129                     | <u>4.66</u> 8        | 9.41 8                         | 4.35 1                 |
| TV-L1-improved [20]       | 6.8  | <u>3.36</u> 2        | 9.63                         | 2.62 2                     | <u>2.82</u> •             | 10.7 :                                | 5 2.23 <mark>3</mark>               | <u>6.50</u> 8           | 15.89                                     | 2.736                                | <u>3.80</u> 5                 | 21.3 11                       | 1.76 2                           | <u>3.34</u> 2        | 4.38 🕻           | 2.39 1                 | <u>5.97</u> 1   | 18.1 1                         | 35.67 11                            | <u>3.57</u> 15          | 4.92 10                        | 3.43 17                   | <u>4.01</u> 7        | 9.84 9                         | 3.44 3                 |
| Multicue MRF [24]         | 7.9  | <u>4.50 </u> 9       | 10.1 3                       | 3 4.18 12                  | <u>2.52</u> 3             | 7.07                                  | 2.36 6                              | 3.09 1                  | 7.41 1                                    | 2.36 3                               | <u>4.46</u> 10                | 20.8 10                       | 2.73 9                           | <u>3.51</u> <b>i</b> | 4.11 2           | 4.06 14                | <u>6.08</u> 9   | 15.66                          | 5.40 9                              | <u>5.25</u> 22          | 5.36 18                        | 9.02 👥                    | <u>3.63</u> <b>i</b> | 8.39 <b></b>                   | 4.15                   |
| F-TV-L1 [18]              | 8.5  | <u>5.44</u> 12       | 12.5 1                       | 1 5.69 16                  | <u>5.46</u> 1             | 15.0 1                                | 3 4.03 13                           | 7.48 13                 | 16.3 <mark>1</mark>                       | 13.42 11                             | <u>5.08</u> 12                | 23.3 15                       | 2.81 10                          | <u>3.42</u> 3        | 4.34 3           | 3.03 3                 | <u>4.05</u> 3   | 15.1 🖡                         | 3.18 2                              | <u>2.43</u> 8           | 3.928                          | 1.87 6                    | <u>3.90</u> 6        | 9.35 7                         | 2.61                   |
| DPOF [21]                 | 10.2 | <u>5.63</u> 13       | 10.9                         | 4.16 11                    | <u>4.05</u> 10            | 12.1                                  | 3.31 9                              | <u>3.87</u> 3           | 8.82 2                                    | 3.17 9                               | <u>4.34</u> 8                 | 16.2 <mark>2</mark>           | 3.13 12                          | <u>3.95</u> 12       | 4.78 1           | 4.17 16                | <u>6.69</u> 15  | 15.2 5                         | 6.27 📭                              | <u>5.62</u> 23          | 6.89 2                         | 6.60 <mark>21</mark>      | <u>2.44</u> 1        | 4.83 1                         | 3.747                  |
| Brox et al. [8]           | 10.2 | <u>4.80</u> 11       | 14.4 1                       | 5 4.29 <mark>13</mark>     | <u>4.05</u> 10            | 13.5 1                                | 03.71 11                            | <u>6.63</u> 10          | 16.0 <mark>1</mark>                       | 7.26 13                              | <u>5.22</u> 13                | 22.7 1                        | 3.22 13                          | <u>4.56</u> 17       | 6.09 🛛           | 3.40 🕻                 | <u>3.97</u> 2   | 17.9 <mark>1</mark> 1          | 1 3.41 <mark>3</mark>               | <u>2.07</u> 3           | 3.76 6                         | 1.18 2                    | <u>5.14</u> 10       | 11.9 12                        | 4.28 9                 |
| Fusion (9)                | 10.7 | <u>4.43</u> 7        | 13.7 1                       | 3 4.08 9                   | 2.47 1                    | 8.91                                  | 2.24 🖡                              | <u>3.70</u> 2           | 9.68 3                                    | 3.128                                | <u>3.68</u> 3                 | 19.87                         | 2.54 8                           | <u>4.26</u> 15       | 5.16 1           | 4.31 18                | <u>6.32</u> 11  | 16.89                          | 6.15 13                             | <u>4.55</u> 19          | 5.78 🗙                         | 3.10 16                   | <u>7.12</u> 18       | 13.6 18                        | 7.86 1                 |
| SegOF [13]                | 11.2 | <u>5.85</u> 14       | 13.5 <mark>1</mark>          | 2 3.987                    | <u>7.40</u> 😢             | 14.9 1                                | 28.13 19                            | <u>8.55</u> 15          | 17.3 📭                                    | 9.01 1 <b>4</b>                      | <u>6.50</u> 16                | 18.1 <mark>3</mark>           | 5.14 16                          | <u>3.90</u> 11       | 4.53 6           | 4.81 21                | <u>6.57</u> 14  | 21.7 1                         | 6.81 17                             | 1.65 t                  | 3.49 5                         | 1.08 1                    | <u>3.71</u> 5        | 9.23 6                         | 3.63                   |
| Dynamic MRF [10]          | 11.2 | <u>4.58</u> 10       | 12.4 1                       | 0 4.14 10                  | <u>3.25</u> 8             | 13.9 <mark>1</mark>                   | 1 2.27 5                            | <u>6.02</u> 7           | 16.8 🛿                                    | 2.36 3                               | <u>4.39</u> 9                 | 22.6 13                       | 2.51 7                           | <u>3.61</u> 5        | 4.551            | 3.46 <mark>5</mark>    | <u>6.81</u> 16  | 22.2 <mark>2</mark>            | 16.78 <mark>16</mark>               | <u>2.41</u> 7           | 3.48 🕻                         | 3.69 18                   | <u>9.26</u> 22       | 17.8 🕿                         | 210.22                 |
| CBF [15]                  | 12.1 | <u>3.95</u> 5        | 10.1                         | 3.44 6                     | <u>3.70</u> 9             | 10.6                                  | 3.85 12                             | <u>5.64</u> 6           | 13.5 5                                    | 3.34 10                              | <u>3.71</u> •                 | 21.5 12                       | 1.99 🕻                           | <u>4.36</u> 16       | 5.50 10          | 3.557                  | <u>11.3</u> 23  | 19.1 🛚                         | <mark>5</mark> 9.05 <mark>22</mark> | <u>6.79</u> 25          | 7.37 🔉                         | 11.6 25                   | <u>5.50</u> 11       | 11.8 11                        | 5.66 1                 |
| GraphCuts [17]            | 13.2 | <u>6.25</u> 15       | 14.3 1                       | <b>4</b> 5.53 15           | <u>8.60</u> 17            | 20.1 1                                | 9 6.61 15                           | <u>7.91</u> 14          | 15.47                                     | 10.9 15                              | <u>4.88</u> 11                | 19.0 5                        | 3.05 11                          | <u>3.78</u> 7        | 4.71 10          | 3.94 12                | <u>8.74</u> 19  | 16.48                          | 5.39 8                              | <u>4.04</u> 18          | 4.87 1                         | 4.85 👥                    | <u>6.35</u> 14       | 12.2 13                        | 6.05 1                 |
| Learning Flow [14]        | 13.2 | <u>4.23</u> 6        | 11.7                         | 3.41 5                     | <u>4.16</u> 12            | 15.3 1                                | <b>4</b> 3.42 10                    | <u>6.78</u> 11          | 16.9 1                                    | 33.83 12                             | <u>6.41</u> 15                | 25.3 17                       | 4.25 💶                           | <u>4.66</u> 20       | 6.01 🛫           | 24.00 13               | <u>6.33</u> 13  | 20.7 1                         | 5.30 6                              | <u>3.09</u> 13          | 4.84 1                         | 2.91 14                   | <u>7.08</u> 17       | 15.0 🗙                         | 5.27 1                 |
| Second-order prior [11]   | 13.5 | <u>3.84</u> <b>i</b> | 11.21                        | 3.11 3                     | <u>3.12</u> 7             | 12.9                                  | 2.17 2                              | <u>6.96</u> 12          | 17.2 1                                    | 2.831                                | <u>3.84</u> 6                 | 20.5 <mark>9</mark>           | 2.09 5                           | <u>4.83</u> 22       | 5.83 🗙           | 3.90 11                | <u>14.0</u> 25  | 21.8 🗙                         | 8.28 19                             | <u>7.74</u> 26          | 6.88 2                         | 11.7 🔉                    | <u>6.74</u> 16       | 13.4 17                        | 5.80 1                 |








































## 00:00:09,751



## 00:00:09,954



## 00:00:00,203





## 00:00:00,208

